



Emperor Penguins Colony: a new metaheuristic algorithm for optimization

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Abstract

A metaheuristic is a high-level problem independent algorithmic framework that provides a set of guidelines or strategies to develop heuristic optimization algorithms. Metaheuristic algorithms attempt to find the best solution out of all possible solutions of an optimization problem. A very active area of research is the design of nature-inspired metaheuristics. Nature acts as a source of concepts, mechanisms and principles for designing of artificial computing systems to deal with complex computational problems. In this paper, a new metaheuristic algorithm, inspired by the behavior of emperor penguins which is called Emperor Penguins Colony (EPC), is proposed. This algorithm is controlled by the body heat radiation of the penguins and their spiral-like movement in their colony. The proposed algorithm is compared with eight developed metaheuristic algorithms. Ten benchmark test functions are applied to all algorithms. The results of the experiments to find the optimal result, show that the proposed algorithm is better than other metaheuristic algorithms.

Keywords Metaheuristic · Optimization · Emperor penguins colony algorithm · EPC algorithm · Optimization techniques · Nature-inspired · Benchmark test functions

1 Introduction

There is no particular algorithm to achieve the best solution for all optimization problems. Also, most algorithms cannot simultaneously provide accuracy and velocity of proper convergence for all optimization problems. So up to now, a variety of nature-inspired algorithms are proposed for optimization. Nature acts as a source of concepts, mechanisms and principles for designing of artificial computing systems to deal with complex computational problems [1].

Optimization is the process of creating something better. In other words, optimization is the mathematical process, the process of adjusting the inputs of a device, or experiment to find the minimum or maximum output or result. Inputs include variables, such as process or function as cost function, target function, fitness function, and outputs include cost and fitness [2].

Most heuristic and metaheuristic algorithms are taken from the behavior of biological systems or physical systems in nature. Metaheuristic algorithms can be defined as high-level methodologies that can be used as a guiding strategy in designing underlying heuristics to solve specific optimization problems [3]. Solving large problems, solving problems faster, and obtaining robust algorithms are three main purposes of modern metaheuristic algorithms to carry out global search [4].

Briefly, the main characteristics of metaheuristic methods are stated below:

1. Unlike heuristic methods, the main purpose of these methods is to find effective and efficient solution instead of finding approximate solution.
2. Metaheuristic methods are policies and strategies that guide the search process.

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3. Metaheuristic methods are approximate and often uncertain (random).
4. Metaheuristic algorithms are used to solve a wide range of optimization problems unlike heuristic methods.
5. Advanced metaheuristic methods save the experience and information obtained during the search process to direct searching to better areas of solution space.

In short, it is possible to say that metaheuristic algorithms are advanced and general strategies for searching, and suggest steps and criteria that are very effective in not staying in local optima trap. An important factor in these methods is the dynamic balance between diversification and intensification strategies. Diversification refers to widespread searches in the solution space, and intensification refers to exploiting the experiences gained in the search process and focusing on better areas of solution space. Therefore, by creating a dynamic balance between these two strategies, the search directs to solution space area, that better solutions are found in this area.

Metaheuristics are generally classified into three categories. These categories are physical-based methods, evolutionary-based methods, and swarm-based methods. In physical-based methods, search agents move across the search space according to the laws of physics such as electromagnetic force, displacement, inertia force, gravity, and so on. In evolutionary-based methods, biological evolution such as reproduction, recombination, selection, and mutation are inspired. Swarm-based methods are based on the collective behavior of social creatures. The collective behavior of social creatures is also combined with a collective intelligence that derives from their environment [5]. Some of the popular techniques are Genetic Algorithms (GA) [6], Differential Evolution (DE) [7], Particle Swarm Optimization (PSO) [8], Ant Colony Optimization (ACO) [9], Simulated Annealing (SA) [10], Cuckoo Search (CS) [11], Bat-inspired Algorithm (BA) [12], Firefly Algorithm (FA) [13], Harmony Search (HS) [14], Tabu Search (TS) [15, 16], Imperialist Competitive Algorithm (ICA) [17], Artificial Bee Colony (ABC) [18], Krill Herd Algorithm (KHA) [19], Invasive Weed Optimization (IWO) [20], Shuffled Frog Leaping Algorithm (SFLA) [21], Intelligent Water Drops (IWD) [22], Grey Wolf Optimizer (GWO) [23], Squirrel Search Algorithm (SSA) [5], Owl Search Algorithm (OSA) [24], Atom Search Optimization (ASO) [25], Salp Swarm Algorithm (SSA) [26], Ant Lion Optimizer (ALO) [27], Whale Optimization Algorithm (WOA) [28], Moth-Flame Optimization (MFO) [29], and Grasshopper Optimization Algorithm (GOA) [30].

In this paper, a new metaheuristic algorithm inspired by the behavior of emperor penguins called Emperor Penguins Colony (EPC) is proposed. The emperor penguins in the colony seek to create the appropriate heat and regulate

their body temperature, and this heat is completely coordinated and controlled by the movement of the penguins.

Rest of the paper is structured as follows: Sect. 2 describes emperor penguins in the nature. Section 3 describes Emperor Penguins Colony (EPC) algorithm. Section 4 includes experimental results and discussion. Section 5 represents conclusions.

2 Emperor Penguins in the nature

The Southern Ocean ecosystem is unique because of the presence of Krill (*Euphausia superba*) that directly or indirectly feeds creatures such as whales, squid, fish, seals, and sea birds such as penguins [31]. Penguins are a species of aquatic birds that cannot fly. Except Galapagos penguin, the rest of the penguin species live exclusively in the southern hemisphere. The number of species are still unknown. But so far about 17–20 species are identified. The Emperor Penguin (*Aptenodytes forsteri*) is the largest species of penguins [32]. The height of a mature emperor penguin is between 110 and 130 cm. Of course, this height is related to the penguin that is walking and extending its neck. The penguin may be less than 80 cm, if stands in the cold, pulls its head in his body and the body supported by tail and intratarsal joints [33]. It is possible that these birds are the best divers among the birds. Figure 1 shows a comparison between heights of different species of penguins that live in Antarctica.

The emperor penguins in the Antarctica are breeding on sea ice [34]. This kind of birds is the only vertebrate species that grows in the harsh winter conditions in Antarctica [35]. In this environment, air temperature may reaches $-40\text{ }^{\circ}\text{C}$. To resistance this harsh environment and energy-saving, the emperor penguins have a significant behavioral, morphological and physiological adaptation [35].

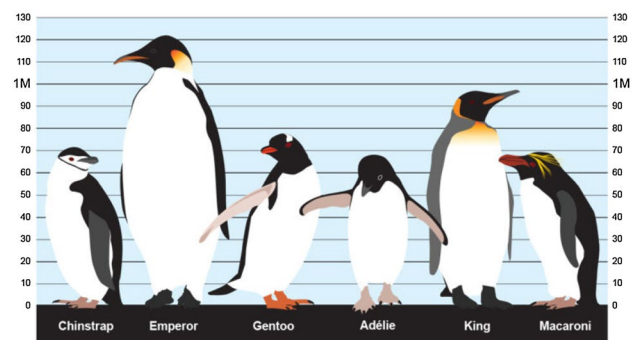


Fig. 1 Heights of different penguin species. Image by Natural Environment Research Council (NERC)/British Antarctic Survey (BAS). Available in: <https://www.bas.ac.uk/>

2.1 Behaviors

The emperor penguin is a totally social animal [36]. These birds reach their full growth within 3 years, and then they can breed. The female penguin lays one egg then transfers the egg to the male penguin due to the end of nutrition reserves and tiredness and goes to the sea for eating about 2 months [37]. The body weight of the male penguin is much higher than the female, and its fat makes it possible to spend the winter and take care of the egg [32]. The egg needs 115 days to become a chick, during this time the penguin is in its colony. To escape from cold males huddle together [38].

2.2 Colonies

In recent decades, coloniality is considered very much, and many researches are done about its evolution and performance [39]. Colonial breeding is widespread especially in birds. About 98% of sea birds grow in their colonies [40]. The emperor penguins have no nests to breed their chicks. They make clusters of thousands penguins called huddles. By forming the huddles, they are protected against cold and wind [41]. Huddling often occurs during chick breeding and in the middle of winter. Unlike other penguin species, only the male emperor penguin take care of the egg by placing the egg on top of the legs and under the abdomen [42]. In a huddle, the body surface temperature of a penguin can reach 37 °C in less than 2 h [37]. The emperor penguins can keep their body temperature high and minimize heat loss [33]. Keeping body temperature high during the incubation period is very important, because the full growth of the fetus requires a temperature about 35 °C [37]. Huddling poses an interesting physical problem [43]. If the density of the huddle is too low, the penguins lose a lot of energy, and if the density is too high, the internal reconstruction becomes impossible, and the penguins that are around cannot easily reach the huddle center which is warmer.

The classic view is that huddles keep their dense form for a few hours or a few days. During this time, the huddle moves slowly. These huddles can be composed of hundreds of penguins and the densities can be reached to 10 birds/m² [37]. Figure 2 shows a sample of emperor penguin huddling group.

2.3 Movement in huddles

As already mentioned, the emperor penguins are a completely social species with no dominance hierarchy. They do not defend any territory and do not attack usually. All individuals benefit the same from huddling for breeding. Huddling behavior of the emperor penguins is a very complicated behavior that is previously described. Birds may



Fig. 2 Emperor penguin huddling group. Picture of the wintering group at Pointe Géologie, which consists of about 2500 males. Available in Gilbert et al. [37]

create huddle many times during the day. Heterogeneity within the general huddling group ensures that everyone becomes warm enough. In this way, the emperor penguins make the coolest environments to the warm environment by huddling together [37].

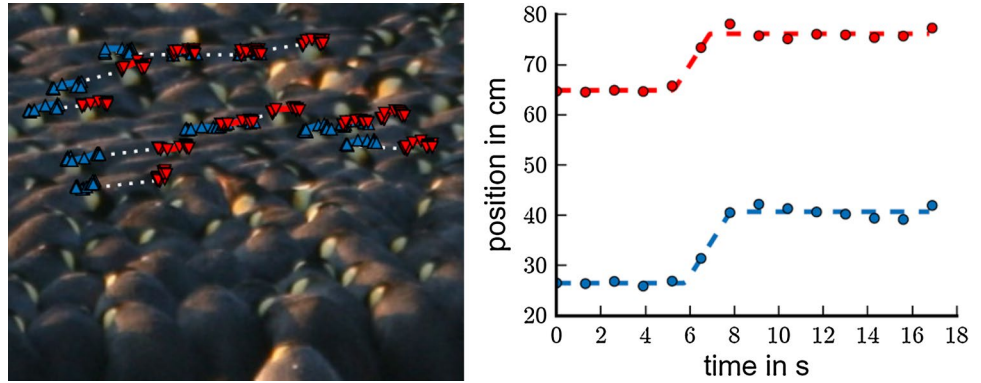
At the center of the huddle, there is much more heat. To use heat by individuals, they do a spiral-like movement toward the center [44]. This spiral-like movement causes the same heat used by individuals. Figure 3 shows a coordinated spiral-like movement in huddle.

Figure 4 shows the markers indicate the positions of the neighboring penguin pairs (connected by white dotted lines). The front penguin of a pair is labeled in red and the rear penguin in blue [35]. Chart in the Fig. 4 shows trajectories from neighboring penguins with similar vertical (y) positions show correlated steps in the horizontal (x) direction. The circles represent the measured positions [35]. These figures and the indicated movement direction in the figures show that there is a kind of coordinated spiral movement in the huddle. This spiral movement makes the heat inside the huddle reach as much as the penguin needs.



Fig. 3 In a colony of penguins spiral-like behavior of the individuals can be observed. Original image is in landscapes & cycles: An environmentalist's journey to climate skepticism by Jim Steele

Fig. 4 Left: Markers indicate the positions of the neighboring penguin pairs. Right: Trajectories from neighboring penguins. Image and graph created by Gerum and Zitterbart and available in paper with title “The origin of traveling waves in an emperor penguin huddle”, published by the open access new journal of physics [35]



3 Emperor Penguins Colony (EPC) algorithm

In this section, the proposed algorithm is described in detail. Before describing the algorithm, we point out that a paper with the same idea has been published [45]. However, that paper algorithm, method, and solution are different from the proposed EPC algorithm in this paper. The proposed method in [45] introduces a mechanism based on the huddle boundary of emperor penguins, which is named the EPO. In that method, the temperature profile around the huddle is calculated, and the algorithm has vector-based equations, while we calculated the body temperature and body heat radiation of each penguin and then due to distance and attractiveness each penguin performs the spiral-like movement. For more details, the proposed algorithm has been described as follows:

3.1 The proposed algorithm

All penguins are scattered throughout the environment. The position of each penguin and its cost are calculated. The cost of penguins are compared with each other. Penguins are always moving towards a penguin that has a low cost (high heat intensity) of absorption. This cost is determined by the heat intensity and the distance. Attraction is done, a new solution is evaluated and the heat intensity is updated. All solutions are sorted and the best is selected. Damping ratio for heat radiation, movement, and heat absorption is applied. Algorithm 1 describes pseudo code of the EPC algorithm. For this algorithm, there are some rules as follows:

- 1) All penguins in the initial population have heat radiation and attract to each other due to absorption coefficient.
- 2) The body surface area of all penguins is considered equal to each other.
- 3) Penguin absorbs the full heat radiation and the effect of the earth's surface and the atmosphere are not regarded.
- 4) The heat radiation of penguins is considered linear.

- 5) The attraction of penguin is done according to the amount of heat in the distance between two penguins. In the longer distance, the less heat is received and in the shorter distance, the more heat is received.
- 6) The penguin spiral movement during the absorption process is not monotonous and has a deviation with uniform distribution.

Algorithm 1: Pseudo code of the Emperor Penguins Colony (EPC) algorithm.

```

generate initial population array of EPs (Colony Size);
generate position of each EP;
generate cost of each EP;
determine initial heat absorption coefficient;
for It=1 to MaxIteration do
    generate repeat copies of population array;
    for i=1 to n population do
        for j=1 to n population do
            if costj < costi then
                calculate heat radiation (Eq. 6);
                calculate attractiveness (Eq. 11);
                calculate coordinated spiral movement (Eq. 18);
                determine new position (Eq. 19);
                evaluate new solutions;
            end
        end
    end
    sort and find best solution;
    update heat radiation (decrease);
    update mutation coefficient (decrease);
    update heat absorption coefficient (increase);
end

```

To calculate the heat intensity and attractiveness, the heat radiation transfer must be calculated. To calculate the heat radiation of each penguin, the calculation of the body surface area of each penguin is needed. The above mentioned quantities are described in subsections. Also, the indirectly movement of penguins is inspired by the coordinated spiral movements of penguins, as described in Sect. 3.5.

3.2 Body surface area

A simple measured model of an emperor penguin, which is standing still in the cold, including main body, head, beak, flippers and feet is represented in Fig. 5 [38, 46].

Through Eq. 1, the main body surface area (prolate spheroid) A_{trunk} (m^2) is obtained.

$$A_{trunk} = 2\pi \frac{ab}{e} \sin^{-1} e + 2\pi b^2, \quad (1)$$

where $e = \sqrt{a^2 - b^2}$ and a is semi-major and b is minor axes lengths. a is half the body length, of the neck where the color of the penguin feathers is white, to the abdomen (0.34 m), b is half the diameter of the main trunk (0.16 m).

Through the cone equation, the beak area is calculated.

$$A_{beak} = \pi r s, \quad (2)$$

where the radius r is the half of the largest sectional area of the beak (0.02 m) and s is the hypotenuse that is considered as the beak length (0.11 m).

Through the sphere equation, head area is calculated. The equation is,

$$A_{head} = \pi d^2 - \pi r^2, \quad (3)$$

where the width (0.11 m) measured from the bottom of black head plumage and r as above and the diameter d is taken to be the mean of head height.

The flippers are considered as a rectangle of length l and width w obtained by Eq. 4.

$$A_{flipper} = l \times w, \quad (4)$$

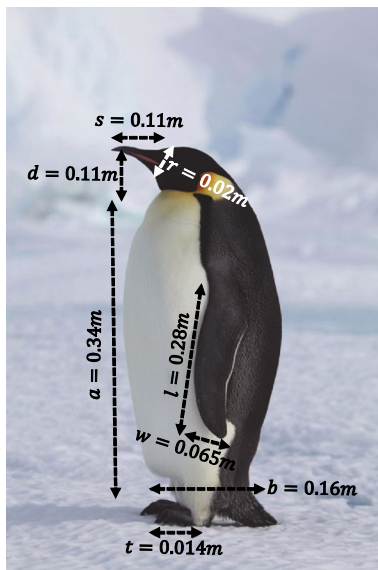


Fig. 5 Measured model of emperor penguin. Original image taken by Stephanie Jenouvrier, Woods Hole Oceanographic Institution

the length l is 0.28 m and the width w is 0.065 m.

By placing each foot on a 1mm² graph paper, the area of each foot is achieved. The main surface area of one foot in contact with the ground is 0.0036 m². Therefore, the total surface of the foot is almost twice the surface measured. The emperor penguins often rest on the tarsometatarsus joint, so this area is also calculated and is 0.0006 m². t is the thickness of the foot and is calculated by the mean thickness of metatarsi (0.014 m).

So, the total body surface area of the emperor penguin becomes 0.56 m². Table 1 describes calculated surface area and percentage of total surface area.

3.3 Heat transfer

Heat transfer is a process function or path function. Therefore, the amount of heat transferred in the thermodynamic process that changes the state of the system depends on how this process occurs and how the initial and final status of the system. The heat transfer is calculated with the heat transfer coefficient and actually calculates the proportion between the heat flux and the thermodynamic heat, that is, the heat transfer is accompanied by the change in the internal energy of the material and always is carried out from the warmer object to the cooler object. Heat flux is a quantitative, vectorial representation of heat-flow through a surface. The thermal equilibrium occurs when the objects involved and their environment reach the same temperature.

Heat transfer is classified into various mechanisms, such as thermal conduction, thermal convection and thermal radiation [47]. Heat conduction is the direct exchange of kinetic energy particles through the boundary between the two systems that occur at the microscopic level. Heat convection occurs when bulk flow of a fluid (gas or liquid) carries heat along with the flow of matter in the fluid. Thermal radiation occurs through a vacuum or any transparent medium (solid or fluid). This energy transfer is done by using photons in electromagnetic waves [48].

In the EPC algorithm the criterion of the heat transfer by radiation is used; And two other criteria, conduction and convection, are neglected. Radiation is a criterion of the attractiveness of penguins. If the distance between a penguin and a

Table 1 Calculated surface area, percentage of total surface area

Body parts	Surface area	
	(m ²)	(%)
Trunk (excluding flippers)	0.471	83.8
Head and Beak	0.040	7.2
Flippers (outside surface only)	0.036	6.5
Feet	0.014	2.5
Total surface area (A_{total})	0.56	100

warmer penguin is less, the warmer penguin is more attractive. In the following the heat transfer by radiation is calculated.

A distributed heat transfer model is considered to estimate the heat exchange for an emperor penguin. In this model, the heat transfer from each body area (trunk, head, flippers and feet) is assembled and it is assumed that the penguin is in a thermal equilibrium with its environment [49],

$$q_{\text{total}} = q_{\text{trunk}} + q_{\text{head}} + q_{\text{flippers}} + q_{\text{feet}}. \quad (5)$$

But the radiation must be calculated. Radiation emitted from each body part of surface area and determined according to,

$$Q_{\text{penguin}} = A\epsilon\sigma T_s^4, \quad (6)$$

where Q_{penguin} is heat transfer per unit time (W), A is total surface area which is calculated in the previous subsection and is 0.56 m^2 . ϵ is emissivity of bird plumage which is considered 0.98 according to [50], σ is the Stefan–Boltzmann constant ($5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4$) and T_s is absolute temperature in Kelvin (K) which is considered $35 \text{ }^\circ\text{C}$ equal to 308.15 K .

3.4 Heat intensity and attractiveness

We know that, in heat transfer by radiation, photons do carry the energy. The photon can be absorbed by increasing rotational or vibrational quantized energy levels, not just electronic energy levels [51, 52]. Photon sources can be homogeneous or concentrated. The latter can be also divided into surface sources, point sources and linear sources [53].

In the surface sources, if the source is I_0 , the equation of receiving the heat photons in I is as follows,

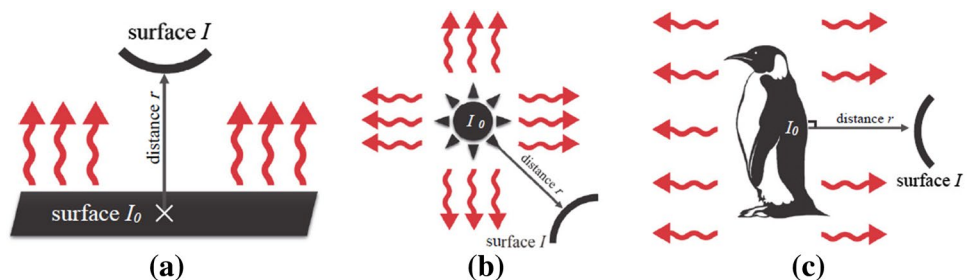
$$I = I_0. \quad (7)$$

In the above equation, since the source area is assumed to be infinite, so the heat source will not be attenuation. Figure 6a is a subjective perception of surface sources.

In the point sources, if the source is I_0 , at distance r , the intensity of the heat received in I is obtained according to Eq. 8. Figure 6b is a subjective perception of point sources.

$$I = \frac{I_0}{r^2}. \quad (8)$$

Fig. 6 **a** Subjective perception of surface sources. **b** Subjective perception of point sources. **c** Subjective perception of linear sources



If the heat source is linear, with the initial value I_0 at distance r , which irradiates vertically on I , the equation of the heat received intensity in I is as follows,

$$I = \frac{I_0}{r}. \quad (9)$$

In this case, the heat attenuation is less than the point heat source. According to the distance between penguins and the type of body physics, in the EPC algorithm, the heat source is considered linear. Figure 6c is a subjective perception of linear sources.

Photons are not slowed down and stopped by matter, but attenuated and scattered. So we need photon attenuation equation that uses linear source Eq. 9. Photon attenuation is,

$$I = I_0 e^{-\mu x}, \quad (10)$$

where μ is attenuation coefficient and x is distance between two linear sources. I_0 is initial heat intensity and I is heat intensity. Finally, the attractiveness Q is defined as,

$$Q = A\epsilon\sigma T_s^4 e^{-\mu x}. \quad (11)$$

3.5 Coordinated spiral-like movements

Figure 7 shows a spiral-like huddle in which the clockwise movement is done around a constant center. In this case, the structure of the system has uncertain boundaries with a spiral pattern around the center.

The temperature is warmest in the center of the huddle and much colder at the perimeter. Penguins do not jostle to gain individual advantage. There is very slow spiral motion of the entire huddle whereby each penguin will have his turn at all positions in the formation [54].

Suppose there are two penguins i and j . Moving always is from the penguin that needs heat to the penguin that is warmer. Here the spiral movement is from i to j , because in this case the penguins j is warmer. (See Fig. 8).

In this paper, the spiral type is considered as a logarithmic spiral. The logarithmic spiral equation is considered as,

$$r = ae^{b\theta}, \quad (12)$$

where θ is the angle of the x -axis, a and b are constant and are selected arbitrary and r shows the distance from the origin. The logarithmic spiral is also known as the equiangular

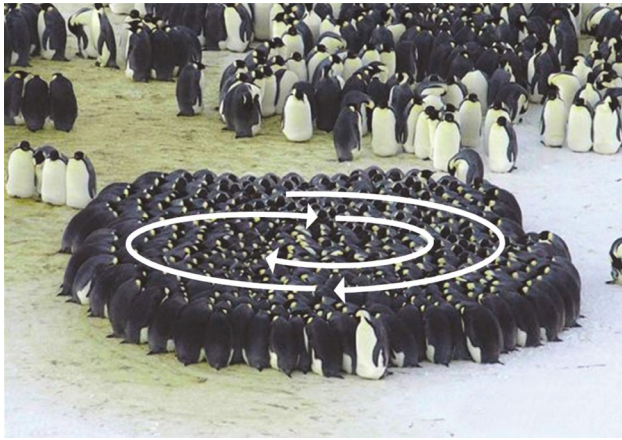


Fig. 7 Spiral-like movement in huddle. Original image taken by Fred <https://www.Olivier/naturepl.com>

spiral, growth spiral, and spiral mirabilis. It can be expressed parametrically as,

$$\begin{aligned} x &= r \cos \theta = a \cos \theta e^{b\theta} \\ y &= r \sin \theta = a \sin \theta e^{b\theta}. \end{aligned} \tag{13}$$

According to [55], the logarithmic spiral can be constructed from equally spaced rays by starting at a point along one ray, and drawing the perpendicular to a neighboring ray. As the number of rays approaches infinity, the sequence of segments approaches the smooth logarithmic spiral.

In Fig. 8, the penguin *i* is attracted into *j* and starts the spiral-like movement. The value of the attractiveness *Q* is a criterion for the rate of distance traveled by the penguin *i* towards *j*. Due to the attractiveness value, it does not reach the destination, and stops after a long distance. *k* is the new position of penguin *i*. This personal experience of spiral-like movement can be used in future moves or can be shared across the entire population. To obtain the equation of this spiral-like movement, first the distance between two penguins *i* and *j* must be calculated according to the equation below,

$$\begin{aligned} D_{ij} &= \int_i^j ds = \int_{\theta_i}^{\theta_j} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2} d\theta \\ &= \int_{\theta_i}^{\theta_j} \sqrt{a^2 b^2 e^{2b\theta} + a^2 e^{2b\theta}} d\theta \\ &= a \sqrt{b^2 + 1} \int_{\theta_i}^{\theta_j} e^{b\theta} d\theta \\ &= \frac{a}{b} \sqrt{b^2 + 1} (e^{b\theta_j} - e^{b\theta_i}). \end{aligned} \tag{14}$$

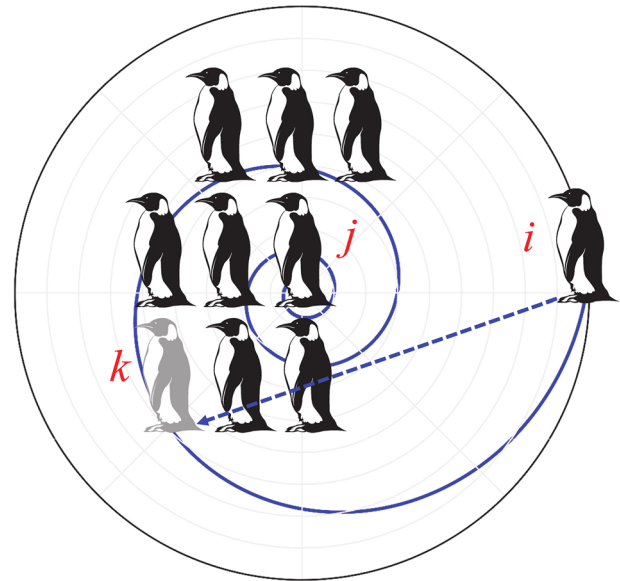


Fig. 8 Subjective perception of emperor penguin coordinated spiral movement

The result is multiplied by attractiveness *Q* to obtain the equation of distance *i* to *k*,

$$\begin{aligned} D_{ik} &= Q \frac{a}{b} \sqrt{b^2 + 1} (e^{b\theta_j} - e^{b\theta_i}) \\ &= \int_i^k ds \\ &= Q \frac{a}{b} \sqrt{b^2 + 1} (e^{b\theta_k} - e^{b\theta_i}). \end{aligned} \tag{15}$$

Now, the relation between Cartesian and Polar coordinates are used. For θ ,

$$\theta = \tan^{-1} \frac{y}{x}, \tag{16}$$

and for x_k and y_k , there are the following equations,

$$\begin{aligned} x_k &= a \cos \theta_k e^{b\theta_k} \\ y_k &= a \sin \theta_k e^{b\theta_k}. \end{aligned} \tag{17}$$

At the end, the *x* and *y* components of the position *k* are obtained in terms of the components *x* and *y* of points *i*, *j*, and attractiveness *Q*. The equation can be considered as follows,

$$\begin{aligned}
 x_k &= ae^{b \frac{1}{b} \ln \left\{ (1-Q)e^{b \tan^{-1} \frac{y_i}{x_i}} + Qe^{b \tan^{-1} \frac{y_j}{x_j}} \right\}} \cos \left\{ \frac{1}{b} \ln \left\{ (1-Q)e^{b \tan^{-1} \frac{y_i}{x_i}} + Qe^{b \tan^{-1} \frac{y_j}{x_j}} \right\} \right\} \\
 y_k &= ae^{b \frac{1}{b} \ln \left\{ (1-Q)e^{b \tan^{-1} \frac{y_i}{x_i}} + Qe^{b \tan^{-1} \frac{y_j}{x_j}} \right\}} \sin \left\{ \frac{1}{b} \ln \left\{ (1-Q)e^{b \tan^{-1} \frac{y_i}{x_i}} + Qe^{b \tan^{-1} \frac{y_j}{x_j}} \right\} \right\}.
 \end{aligned}
 \tag{18}$$

Because the angle information is pre-determined, the spiral-like movement may become monotonous. It's better not to be limited to monotonous spiral path. So a random component is needed to increase diversity. In this way, the penguin i will move spirally then it's summed with a random vector and is transported to a new position. The equation can be considered as follows,

$$Eq. 18 + \varphi \epsilon_i, \tag{19}$$

where φ is the mutation factor in the change of path and ϵ is a random vector. In this way, the coefficient of a random vector is added. ϵ can be Uniform, Normal or Lévy distribution. In the EPC algorithm a Uniform distribution is used. The function of this equation is exactly the same as the function of a mutation, which is performed after the crossover in the GA algorithm.

4 Experimental results and discussion

In this section the details of conducting the experiment to evaluate the efficiency of the EPC algorithm are described. For evaluating the performance of the proposed algorithm, ten standard benchmark test functions are used. The results of the algorithm performance evaluation are compared with eight well-known metaheuristic algorithms.

4.1 Benchmark functions

Test functions are known as artificial landscapes in applied mathematics. These functions are very useful for evaluating the characteristics of optimization algorithms. Performance, accuracy, overall performance, and convergence rates are some of the characteristics that can be evaluated with these functions. These functions are listed below [56]:

Ackley function: Usually, most authors use Ackley function in their optimization algorithm experiments [57]. This function has many local minima and places the risk of being trapped in one of these local minima for an optimization algorithm [58]. The Ackley function is,

$$\begin{aligned}
 f(x) &= -ae \left(-b \sqrt{\frac{1}{d} \sum_{i=1}^d x_i^2} \right) - e \left(\frac{1}{d} \sum_{i=1}^d \cos(cx_i) \right) \\
 &+ a + e(1),
 \end{aligned}
 \tag{20}$$

where $a = 20$, $b = 0.2$ and $c = 2\pi$ are recommended variable values. For all $i = 1, \dots, d$ the function is evaluated on the hypercube $x_i \in [-32.768, 32.768]$, although it may be restricted to a smaller domain. At $x^* = (0, \dots, 0)$ the global minimum is $f(x^*) = 0$ [59].

Sphere function: Except for the global one the Sphere function has d local minima [58]. It is unimodal, convex and continuous. For all $i = 1, \dots, d$, the function is evaluated on the hypercube $x_i \in [-5.12, 5.12]$. The Sphere function is [60],

$$f(x) = \sum_{i=1}^d x_i^2, \tag{21}$$

where at $x^* = (0, \dots, 0)$, the global minimum is $f(x^*) = 0$.

Rosenbrock function: For gradient-based optimization algorithms the Rosenbrock function is a popular test problem. The global minimum lies in a narrow valley and the function is unimodal [58]. Convergence to the minimum is difficult even if this valley is easy to find. For all $i = 1, \dots, d$ the function is usually evaluated on the hypercube $x_i \in [-5, 10]$, although for all $i = 1, \dots, d$ it may be restricted to the hypercube $x_i \in [-2.048, 2.048]$. The function is [60],

$$f(x) = \sum_{i=1}^{d-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right], \tag{22}$$

where at $x^* = (1, \dots, 1)$, the global minimum is $f(x^*) = 0$.

Rastrigin function: This function has several local minima. Although locations of the minima are regularly distributed, but it is highly multimodal [61]. For all $i = 1, \dots, d$ the function is usually evaluated on the hypercube $x_i \in [-5.12, 5.12]$. The Rastrigin function is,

$$f(x) = 10d + \sum_{i=1}^d [x_i^2 - 10 \cos(2\pi x_i)], \tag{23}$$

where at $x^* = (0, \dots, 0)$, the global minimum is $f(x^*) = 0$.

Griewank function: The Griewank function has many widespread local minima, which are regularly distributed [58]. For all $i = 1, \dots, d$ the function is usually evaluated on the hypercube $x_i \in [-600, 600]$. The Griewank function is,

$$f(x) = \sum_{i=1}^d \frac{x_i^2}{4000} - \prod_{i=1}^d \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1, \tag{24}$$

where at $x^* = (0, \dots, 0)$, the global minimum is $f(x^*) = 0$.

Bukin function: The Bukin function has many local minima, all of which lie in a ridge [56]. The Bukin function is usually evaluated on the rectangle $x_1 \in [-15, -5]$, $x_2 \in [-3, 3]$. The function is,

$$f(x) = 100\sqrt{|x_2 - 0.01x_1^2|} + 0.01|x_1 + 10|, \tag{25}$$

where at $x^* = (-10 \dots, 1)$, the global minimum is $f(x^*) = 0$.

Bohachevsky functions: The Bohachevsky functions all have the same similar bowl shape [56]. For all $i = 1, 2$ the functions are usually evaluated on the square $x_i \in [-100, 100]$. The Bohachevsky function is,

$$f(x) = x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) - 0.4 \cos(4\pi x_2) + 0.7, \tag{26}$$

where at $x^* = (0, 0)$, the global minimum is $f(x^*) = 0$.

Zakharov function: Except the global one The Zakharov function has no local minima [56]. For all $i = 1, \dots, d$ the function is usually evaluated on the hypercube $x_i \in [-5, 10]$. The Zakharov function is,

$$f(x) = \sum_{i=1}^d x_i^2 + \left(\sum_{i=1}^d 0.5ix_i\right)^2 + \left(\sum_{i=1}^d 0.5ix_i\right)^4, \tag{27}$$

where at $x^* = (0, \dots, 0)$, the global minimum is $f(x^*) = 0$.

Booth function: For all $i = 1, 2$ the Booth function is usually evaluated on the square $x_i \in [-10, 10]$. The function is [56],

$$f(x) = (x_1 + 2x_2 + 7)^2 + (2x_1 + x_2 + 5)^2, \tag{28}$$

where at $x^* = (1, 3)$, the global minimum is $f(x^*) = 0$.

Michalewicz function: The Michalewicz function is multimodal and it has $d!$ local minima. The parameter m in this function defines the steepness of they valleys and ridges; a larger m leads to a more difficult search [58]. The recommended value of m is $m = 10$. For all $i = 1, \dots, d$ the function is usually evaluated on the hypercube $x_i \in [0, \pi]$. The function is,

$$f(x) = -\sum_{i=1}^d \sin(x_i) \sin^{2m}\left(\frac{ix_i^2}{\pi}\right), \tag{29}$$

where the global minimum at $d = 2$ is $f(x^*) = -1.8013$, at $x^* = (2.20, 1.57)$.

4.2 Algorithms used for comparisons

To validate the performance of the proposed algorithm, the eight well-known algorithms are chosen for comparison. These are Genetic Algorithm (GA) [6], Imperialist Competitive Algorithm (ICA) [17], Particle Swarm Optimization (PSO) [8], Artificial Bee Colony (ABC) [18], Differential Evolution (DE) [7], Harmony Search (HS) [14], Invasive Weed Optimization (IWO) [20], and Grey Wolf Optimizer (GWO) [23].

4.3 Settings for algorithms

In order that, the experiment results be comparable, the settings of all algorithms are similar to each other. The number of iteration in each run is considered 100. The initial population is considered 20 in all algorithms. Also, the number of decision variables for each algorithm is 5.

The experiment process is as follows: Each algorithm was run 30 times, as mentioned, each run has 100 iterations. So the mean of 30 results was considered. The standard deviation of 30 runs was also calculated.

4.4 Results and discussion

Table 2 shows the mean of 30 runs in using benchmark functions. The table shows that the proposed algorithm in using of functions Ackley, Sphere, Rastrigin, Griewank, Bohachevsky, Zakharov, Booth, and Michalewicz has the best performance. Also, in using of functions that do not have the best performance, its performance is acceptable. The results show that in the case of the Sphere function, which is a simple function without local minima, the EPC algorithm gives the best answer, and then the GWO algorithm has a better answer. The same result is obtained for the Zakharov function, which is a function without a local minima. For functions such as Ackley, Rastrigin, Griewank and Michalewicz, which have several local minima, the best answer is obtained by the EPC algorithm. The Michalewicz function is a multi-modal and complex function, but the best answer is obtained by the EPC algorithm, that its answer is very close to the global minimum. After the EPC algorithm, the GWO algorithm gives a better answer in Michalewicz function. The results show that due to the variety of experiments, the proposed algorithm is suitable for processing under simple conditions without local minima and complex conditions with local minima. Table 3 shows the standard deviation of each algorithm. So, the overall performance of the proposed algorithm is acceptable.

The EPC algorithm has a memory, so that knowledge of good solutions is maintained by all penguins. In this algorithm, each penguin benefits from its past information; while this feature does not exist in the genetic algorithm

Table 2 Mean of 30 runs in using benchmark functions

Functions	Algorithms								
	GA	ICA	PSO	ABC	DE	HS	IWO	GWO	EPC
Ackley	0.0907	2.6265e-04	6.0895e-04	0.1421	1.7711e-04	2.0935	0.0019	1.9177e-05	3.1821e-08
Sphere	0.0028	4.9481e-06	9.0413e-08	0.0102	1.3381e-08	0.4327	9.2105e-07	2.7557e-12	3.3231e-16
Rosenbrock	2.6388	3.3888	1.7793	15.4923	1.9891	69.4738	9.4521	1.7491	3.8752
Rastrigin	0.9373	1.2083	2.9004	10.7627	0.2993	7.3835	14.9577	2.6374	5.8027e-14
Griewank	0.0437	0.0264	0.0222	0.1333	0.0139	0.0444	0.0435	0.0542	0.0131
Bukin	1.1704	0.0759	0.2298	0.7341	0.8872	1.9679	0.2859	0.1509	0.0956
Bohachevsky	0.0085	3.6398e-13	4.9679e-10	3.0094e-07	8.4519e-13	0.0056	2.0029e-07	7.1826e-10	1.1102e-17
Zakharov	2.2252	0.3906	4.7044e-06	3.9614	0.1830	4.4355	2.4909e-06	1.6946e-08	5.4928e-16
Booth	0.0582	0.0028	8.4872e-11	8.1022e-06	2.7995e-05	0.0160	2.7585e-08	1.9738	7.3392e-18
Michalewicz	-4.5633	-4.5706	-4.1158	-2.8306	-4.7952	-4.4896	-3.9391	-2.4935	-1.8030

The best value obtained by algorithms in each row are in bold

Table 3 Standard deviation of 30 runs in using benchmark functions

Functions	Algorithms								
	GA	ICA	PSO	ABC	DE	HS	IWO	GWO	EPC
Ackley	0.0768	3.2035e-04	4.6819e-04	0.0787	8.0917e-05	0.6405	5.7446e-04	5.8496e-05	6.7838e-09
Sphere	0.0048	2.5028e-05	1.3032e-07	0.0066	1.4491e-08	0.3579	5.2142e-07	1.2136e-11	1.3588e-16
Rosenbrock	1.9906	6.8807	1.3998	7.8228	1.2408	63.2841	28.9087	2.0407	0.0435
Rastrigin	0.6570	1.3956	2.0537	3.0770	0.6395	2.2297	7.4386	3.7391	2.5794e-14
Griewank	0.0359	0.0125	0.0140	0.0410	0.0144	0.0114	0.0410	0.1698	0.0118
Bukin	2.7639	0.0509	0.1049	0.4065	0.5171	1.0097	0.1246	0.0922	0.0250
Bohachevsky	0.0358	1.0303e-12	9.8150e-10	5.0249e-07	1.4934e-12	0.0087	2.8608e-07	2.2167e-09	4.4694e-17
Zakharov	3.6876	0.6461	9.7591e-06	2.6351	0.1501	3.2684	1.3890e-06	6.4802e-08	1.8226e-16
Booth	0.1223	0.0147	1.3757e-10	1.4857e-05	4.3622e-05	0.0350	2.6599e-08	0.1435	6.4746e-18
Michalewicz	0.2850	0.3133	0.4863	0.2111	0.0683	0.1596	0.5181	0.4395	0.2612

The name of proposed algorithm is in bold

and the previous knowledge of the problem disappears with the change in population. In the proposed algorithm, each penguin changes its position according to its personal experience of receiving heat and the experiences of the entire community. This feature also exists in the PSO algorithm, so that the population share their own information with each other. Using the information sharing feature among population, the proposed algorithm converges at a very high speed in finding a global optimal solution. Also, with regard to swarm intelligence and optimization strategy, with increasing number of penguins, it is more flexible against the local optimal problem. However, it begins process, with much smaller population in comparison with genetic algorithm.

The EPC algorithm can effectively deal with multi-modal and nonlinear optimization problems. Unlike the PSO, the proposed algorithm does not use velocity and therefore, there is no premature convergence. The EPC algorithm has the potential for integration with other optimization

algorithms and does not require a good initial solution to start its process. Figure 9 shows variation in performance for benchmark functions.

In order to analyze the behavior of the EPC algorithm, it has been run with different population sizes (colony sizes) and different number of decision variables. In Table 4, the mean of best function values with different colony sizes varying as 10, 30, and 50 have been presented.

The result of Table 4 is that, with the colony size between 20 and 30 and number of decision variables between 3 and 10, the proposed algorithm has the best performance. Of course, the algorithm performance with a large colony sizes is acceptable if the number of decision variables is low. We once again performed the experiment with increasing dimension. We considered the population sizes and decision variables as high numbers. Some algorithms fail to find the solutions by increasing the dimension.

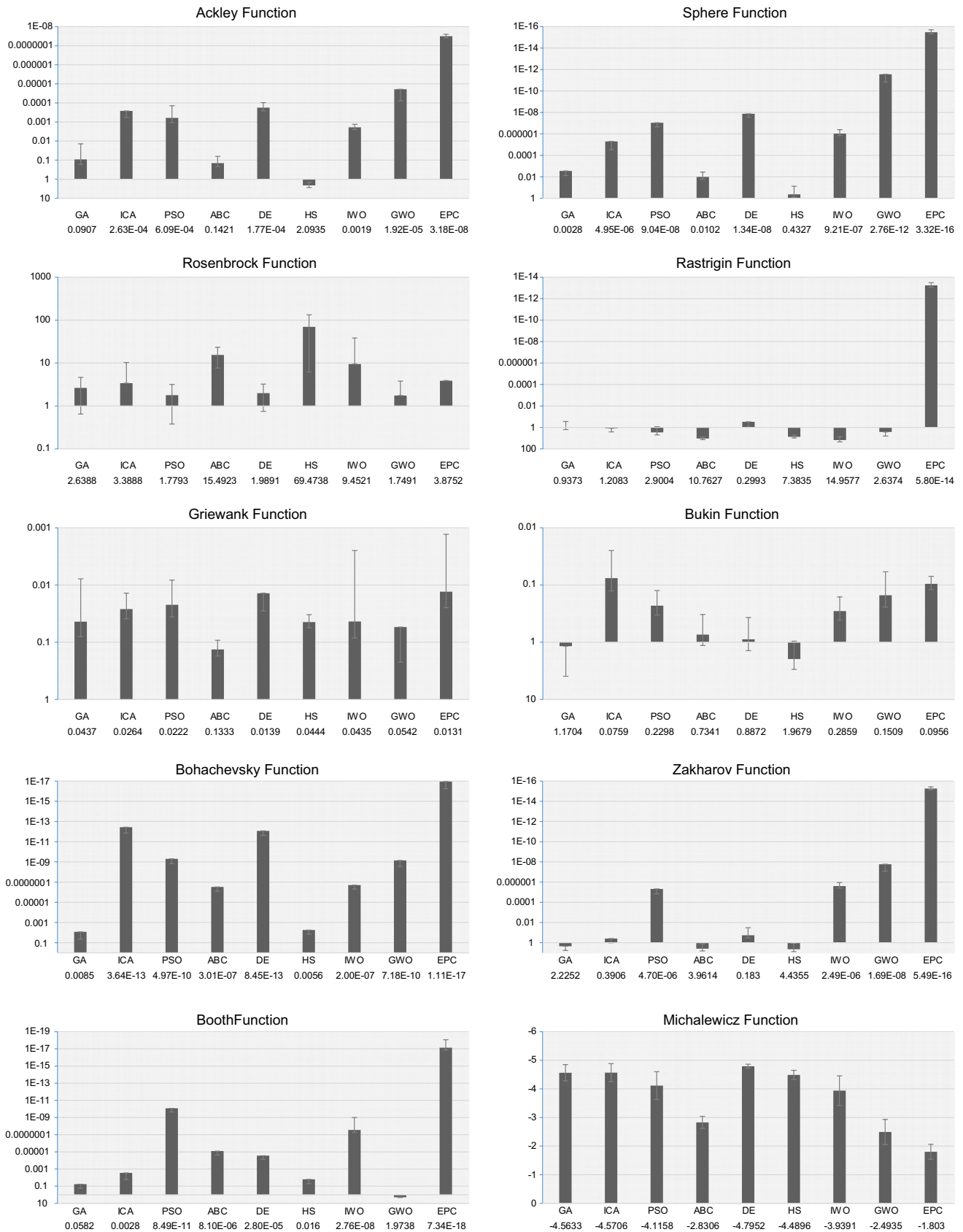


Fig. 9 Variation in performance for benchmark functions

Table 4 Mean of best function values obtained for 300 iterations by EPC algorithm under different colony sizes

Functions	Colony size/Number of decision variables							
	10/3	10/10	30/3	30/10	30/20	50/5	50/20	50/30
Ackley	0	3.0964	0	0.0385	0.7608	0	3.2611e-04	0.3296
Sphere	0.0169	2.7661	1.4094e-45	8.3873e-44	0.1539	8.3867e-45	3.7835e-43	0.0287
Rosenbrock	9.4344	1.0586e+03	0.0504	9.3333	61.6074	0.9358	18.8678	61.7444
Rastrigin	1.7764e-14	4.3343e-13	0	3.2401e-13	9.5821e-13	2.9132e-14	8.7791e-13	1.6769e-12
Griewank	0.0086	0.3059	0	0	0.0237	0.0036	2.6302e-04	0.0073
Bukin	0.1079	0.0965	0.1010	0.0945	0.0987	0.0989	0.0977	0.0935
Bohachevsky	1.2212e-16	1.4433e-16	0	0	0	0	0	0
Zakharov	0.6459	23.7024	1.9280e-45	0.2126	9.4870	1.3624e-44	1.4947	7.4937
Booth	0	0	0	0	0	0	0	0
Michalewicz	- 2.5055	- 5.6178	- 2.7947	- 6.9287	- 11.0801	- 4.5589	- 12.0014	- 15.5106

Table 5 Mean of best function values obtained for 100 iterations by EPC algorithm under different colony sizes with high dimensions

Functions	Colony size/Number of decision variables					
	100/50	100/100	500/250	500/500	1000/500	1000/1000
Ackley	0.1311	0.7872	3.7043	7.4912	7.0212	10.1354
Sphere	0.0308	0.7395	8.0030	14.1387	11.4389	72.5916
Rosenbrock	48.8622	50.4598	139.8044	146.2557	143.9585	149.5585
Rastrigin	3.1264e-12	3.1554e-11	2.2336e-10	2.3311e-10	3.2256e-09	3.5654e-09
Griewank	0.6487	1.1116	1.4209	1.6501	1.5365	1.8579
Bukin	0.1001	0.1001	0.1000	0.1004	0.1002	0.1005
Bohachevsky	0	0	0	2.1147e-08	2.6632e-07	3.1719e-07
Zakharov	10.3383	12.4809	42.8094	45.6094	50.1223	53.2556
Booth	1.8235e-08	2.1242e-08	1.2396e-09	1.0810e-09	3.1689e-10	7.6893e-10
Michalewicz	- 7.1370	- 9.5135	- 27.9260	- 28.9260	- 18.3652	- 32.6665

Table 5 shows that the proposed algorithm does not fail to find solutions with large dimension. The solutions may not have high quality, but with increasing the iteration, the better solution can be achieved. The mean of best function values with different colony sizes varying as 100, 500, and 1000 have been presented. Figures 10, 11 and 12 show the emperor penguin population in consequent iterations.

To find significant differences between the results obtained by algorithms, statistical analysis is used. To detect significant differences in the results, Friedman and Iman-Davenport tests are employed [62, 63]. Table 6 shows the ranking of algorithms based on the results of Table 2 using the Friedman test. As expected, the EPC algorithm is first in the ranking, then the DE algorithm is located. In the next ranks, the algorithms are PSO, ICA, GWO, IWO, GA, ABC and HS, respectively. Table 7 shows the results of the Friedman and Iman-Davenport tests. In this table, there is the Chi-Square value with 8 degrees of freedom, and also there is asymptotic significance of the test (p-value) with very close to zero value. Being close to zero value of the asymptotic significance, the hypothesis is rejected. Therefore, it can be concluded

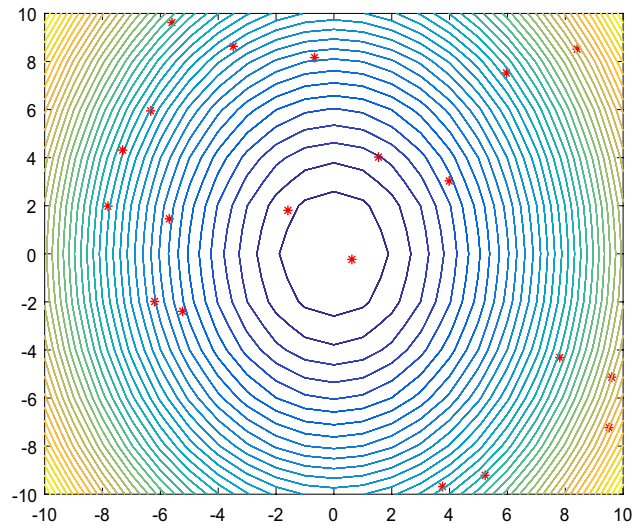


Fig. 10 Convergence in 2nd iteration

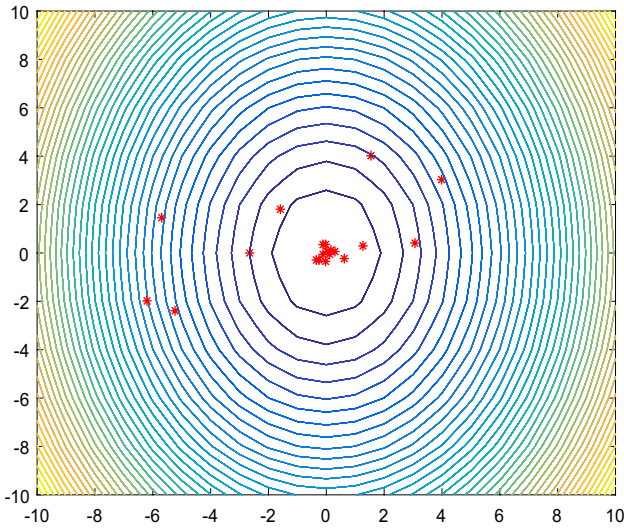


Fig. 11 Convergence in 3rd iteration

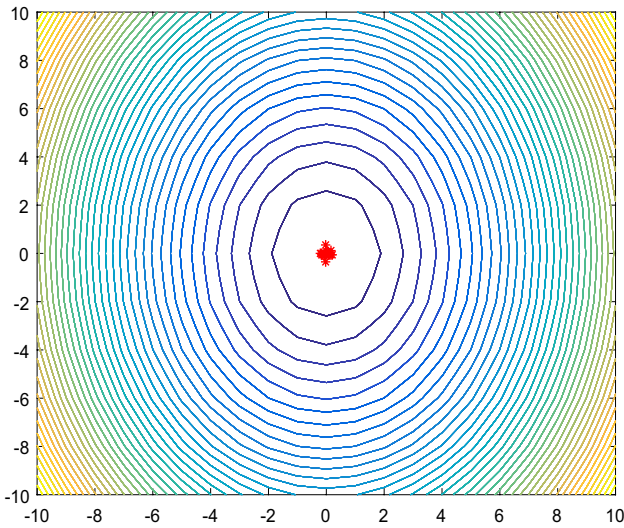


Fig. 12 Convergence in 4th iteration

that there is a significant difference in the performance of algorithms.

As discussed, Table 6 compares the proposed algorithm with all algorithms in the low dimensions. To compare algorithms in high dimensions, we select the top three algorithms from Table 6 based on ranking. In this way,

Table 6 Ranking of algorithms based on performance using Friedman’s test

	Algorithms								
	GA	ICA	PSO	ABC	DE	HS	IWO	GWO	EPC
Ranking	6.20	4.00	3.90	7.30	3.40	7.80	5.50	4.50	2.40

The best rank obtained by Friedman’s test is in bold

Table 7 Results of Friedman’s and Iman–Davenport’s tests based on performance

Test method	Chi-Square	Degrees of freedom (DF)	p value	Hypothesis
Friedman	35.4670	8	2.20e-05	Rejected
Iman–Davenport	7.1677	8	6.12e-07	Rejected

the EPC, DE and PSO are selected for comparison with the high dimensions. Table 5 shows the results of applying the EPC algorithm on test functions in high dimensions. Table 10 in the Appendix A also shows the results of applying the PSO and DE algorithms on test functions for 100, 500 and 1000 dimensions, and the results of various test functions are presented.

Friedman ranking was considered separately for 100, 500 and 1000 dimensions and once again, the overall ranking was done. The ranking results are presented in Table 8. The results show that the DE algorithm does not have a good performance in comparison with the both EPC and PSO algorithms. The EPC algorithm has the first ranking, then the PSO and DE algorithm are located, respectively.

As Table 2 shows, DE has good performance in low dimensions. It has excellent convergence speed and exploration abilities. But in high dimensions, DE has lower convergence speed than PSO and EPC. Table 10 in the Appendix A shows that DE does not provide acceptable results for functions such as Sphere, Rosenbrock, Zakharov, and Michalewicz. DE has difficulties with Rosenbrock function, especially. The results show that in high dimensions, DE cannot solve them. It seems that the differential evaluates to zero in the high dimensions before reaching the optimum. PSO has good convergence speed. It has good exploration abilities. But the EPC performance is much better. EPC has good performance on all explored test functions. In high dimensions, PSO has low convergence speed for most test functions than EPC. The EPC has excellent exploration abilities. EPC has rich potential for local search, global exploration, quick convergence and wide divergence. The penguins have abilities and the algorithm sets their behavior adaptively during the optimization process. During the optimization process, the penguins can explore and learn problems and decide how to behave based on their own learning.

Table 8 Ranking of algorithms based on performance for 100, 500 and 1000 dimensions using Friedman's test

Ranking criteria	Algorithms		
	PSO	DE	EPC
If the dimension is 100	1.80	2.60	1.60
If the dimension is 500	1.80	2.60	1.60
If the dimension is 1000	1.80	2.50	1.70
Total	1.80	2.57	1.63

The name of proposed algorithm is in bold

Table 9 Results of Friedman's and Iman–Davenport's tests based on performance in high dimensions

Test method	Chi-Square	Degrees of freedom (DF)	<i>p</i> -Value	Hypothesis
Friedman	14.8667	2	5.91e-04	Rejected
Iman–Davenport	9.5524	2	2.59e-04	Rejected

Table 9 shows the results of the Friedman and Iman–Davenport tests for high dimensions. In this Table the *p*-value is very close to zero. Being close to zero value of the asymptotic significance, the hypothesis is rejected and it can be concluded that there is a significant difference in the performance of algorithms in high dimensional problems.

The Free Search (FS) algorithm [64] is also one of the algorithms that works well in high dimensional problems. In [65] the authors claim that FS algorithm is better than PSO and DE. For example, the FS algorithm has been tested in 100 dimensions, in all 320 tests on Michalewicz function, successful result has been achieved. Considering the differences in the way in which tests are performed and the differences in algorithm settings, we cannot correctly compare the results in their paper with the results obtained in this paper. Therefore, a careful study and comparison of FS algorithm and proposed EPC algorithm can be the subject of further researches.

In comparison with the similar algorithm in [45], the proposed EPC algorithm has the following advantages: Comprehensive investigation of the emperor penguins lifestyle and

the extraction of mathematical and physical relations; Using the spiral like movement to optimize temperature without need to determine the boundary of the huddle; Using the past information by individuals; Quick convergence; Possibility of starting optimization process with small population; Not limited to monotonous spiral path; Using efficient features of other algorithms such as PSO; Integration ability with other optimization algorithms; acceptable performance to solve high-dimensional problems.

5 Conclusions

In this paper, a new metaheuristic algorithm based on the behavior of emperor penguins in their colonies called Emperor Penguins Colony (EPC) was proposed. The proposed algorithm is a swarm-based and nature-inspired algorithm that is controlled by the thermal radiation and spiral-like movement of penguins. The proposed algorithm is a fundamental algorithm for optimization in the broad area of knowledge such as engineering and science. This algorithm can be improved with inspiring other penguin's behaviors in their colonies. However, the results of the experiments indicate the acceptable performance of this algorithm.

As future work, we consider improving the algorithm and introducing its applications and using it in solving various problems such as clustering problem and community detection in complex networks.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix A

The results of applying the PSO and DE algorithms on test functions for 100, 500 and 1000 dimensions.

See Table 10.

Table 10 Mean of best function values obtained for 100 iterations by PSO and DE with high dimensions

Functions	Algorithms					
	PSO			DE		
	Colony size/Number of decision variables			Colony size/Number of decision variables		
	100/100	500/500	1000/1000	100/100	500/500	1000/1000
Ackley	3.2273	5.5184	6.3592	7.0413	9.8028	10.1344
Sphere	11.9290	46.4180	164.3003	23.4364	346.2924	753.5743
Rosenbrock	29.5721	457.5049	924.0653	2636.5671	10817.9972	11490.8309
Rastrigin	5.8024	9.3882	10.1587	11.3105	15.6018	16.5441
Griewank	0.1860	0.9428	1.3703	1.0468	1.8653	2.8978
Bukin	0.2631	0.1102	0.1370	0.9233	0.4351	0.3147
Bohachevsky	6.4410e-08	1.9020e-08	6.4049e-09	8.4324e-09	5.9890e-09	3.1083e-09
Zakharov	55.9304	370.5257	849.8913	79.1155	428.4201	876.6740
Booth	1.4053e-08	7.7046e-09	2.6212e-09	0.0004	9.5099e-05	6.8868e-05
Michalewicz	-12.8681	-30.8056	-42.8628	-23.1648	-52.4540	-74.6652

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